

# Evaluating mathematical applications within several fields of engineering

Yashovardhan Raniwala

[yasho.raniwala@gmail.com](mailto:yasho.raniwala@gmail.com)

Neerja Modi School,

Jaipur, Rajasthan

## ABSTRACT

*There is extensive use of mathematics and applied mathematics in various engineering fields. Within this paper, numerous illustrations of applications of mathematics in diverse engineering fields including chemical, mechanical and electrical are explored. These discussed applications are actual ones observed in engineering fields in the real world and may differ from those displayed in mathematics textbooks. The objective of this paper is to establish a link between engineering and mathematics and underscore their interconnectedness. It is observed that engineering students face hardships when working on questions requiring extensive use of mathematics. This can be attributed to the fact that although these students have prior knowledge of mathematical concepts including calculus, numerical analysis, and cubic algebra, they are unable to apply it while working on engineering questions. I hope that through these examples, engineering students are encouraged to further their understanding of engineering problems. It is suggested that mathematics professors assign mathematics problems to students who are more directly linked to engineering fields.*

**Keywords**— Applications, Mechanical, Chemical, Electrical

## 1. INTRODUCTION

The foundation of every engineering field is mathematics. Hand-in-hand with physics, mathematics has aided the development of engineering. The rate of progress that engineering fields have experienced would not be possible in absence of mathematics. The lack of mathematics would also reduce the allure and fascination that engineering holds.

Engineering students are taught linear algebra, calculus, statistics and numerical analysis due to their necessity in understanding engineering subjects including fluid mechanics, heat flow dynamics, electrical circuits and property of materials among others. However, several complaints have been raised by students who find relating mathematics to engineering challenges. As an example, they might be taught differential equations, an integral part of the calculus, and then told to apply what they learned to questions relating to heat transfer. The reality for most students is, however, different as they find this application hard. This issue is further aggravated by the mathematics textbooks of engineering students which have superficial explanations and tend to set unlikely assumptions. Refer to [9] for a textbook that has a comprehensive discussion of the real-life applications of mathematical modeling. The aim of this paper is to demonstrate the applications of mathematics in several engineering fields. These applications do not include sophisticated mathematics to make them graspable to a larger audience. The questions have been solved using Maple which is a symbolic programming language. For a basic introduction to Maple, refer to [1 and 13]. To gain a deeper understanding of Maple, refer to solved Maple problems given in [12]. Three questions are presented in the paper from mechanical, chemical and electrical engineering respectively. One is related to beam reflection, other on modeling the state of refrigerants using equations while the last is an illumination question.

## 2. BEAM DEFLECTION FOR A CANTILEVER WITH VARIABLE CROSS-SECTION

Such types of situations are relevant in the evaluation of materials in mechanical, civil and aeronautical engineering. In figure 1, a cantilever beam having a variable cross-section is displayed. At its free end, a concentrated load  $P$  is situated. The presence of this load  $P$  causes the beam to deflect. Our task is to find the curve of deflection along the beam and determine when it is maximum. The relation between the deflection  $Y$  and bending moment  $M_x$  can be written as

$$M_x = EIY^n \quad (1)$$

The above equation, Eq. (1), is found in almost all books related to structural engineering and the strength of diverse materials; refer to [2-4] for examples. In this equation, the other two variables,  $E$  and  $I$  respectively represent the young modulus of elasticity and the beam's moment of inertia. Jointly,  $EI$  is the flexural rigidity of the beam.

While in Eq. (1), any positive answer is considered as the downward deflection, it can also be negative as used in [2]. I (moment of inertia) is positive in case the direction is clock-wise. In order to solve this equation, Eq. (1) can be written as

$$y^n = \frac{M_x}{EI} \tag{2}$$

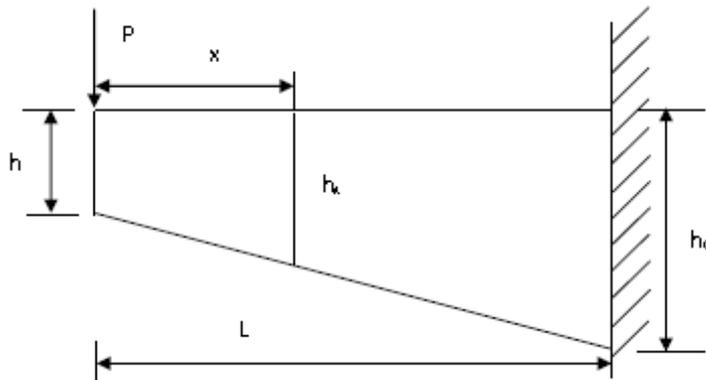


Fig. 1: Beam deflection for a cantilever with variable cross-section

To get the point-wise deflection along the beam, we will have to double integrate Eq. (2). The bending moment for the cantilever shown above in figure 1 can be represented as

$$M_x = -P_x \tag{3}$$

Here x refers to the horizontal distance from the left end of the beam. For the same beam, the moment of inertia can be represented as

$$I_x = \frac{1}{12} bh^2$$

$$h_x = h + (h_0 - h) \frac{x}{L}$$

Where,

$$I_x = \frac{1}{12} b \left[ h + (h_0 - h) \frac{x}{L} \right]^2 = \frac{1}{12} b \left[ h + h_0 \left( 1 - \frac{h}{h_0} \right) \frac{x}{L} \right]^2$$

Giving this can also be equated as

$$I_x = \frac{1}{12} bh^2 [m + (1 - m)p]^2 = I_0 [m + (1 - m)p]^2$$

$$I_0 = \frac{1}{12} bh^2$$

Where,

$$m = \frac{h}{h_0}, 0 < m \leq 1 \quad \text{and} \quad p = \frac{x}{L}, 0 \leq p \leq 1$$

Thus Eq. (2) can be written as

$$y^n = - \frac{P_x}{EI_0 [m + (1 - m)p]^2} = - \frac{P_p L}{EI_0 [m + (1 - m)p]^2} \tag{4}$$

The dimensional aspect from Eq. (4) can be removed by dividing it by PL/EI<sub>0</sub>. This then can be represented as

$$y^n = \left( \frac{EL_0}{PL} \right) = Y(p) = - \frac{P_p L}{EI_0 [m + (1 - m)p]^2} \tag{5}$$

By double integrating Eq. (5) we get

$$Y'(p) = \frac{1}{(1 - m)^2} \left[ - \frac{1}{2} \frac{m}{[m + (1 - m)p]^2} + \frac{1}{[m + (1 - m)p]} \right] + C_1 \tag{6}$$

$$Y''(p) = \frac{1}{(1 - m)^2} \left[ \frac{1}{2} \frac{m}{[m + (1 - m)p]} + \ln[m + (1 - m)p] \right] + C_1 p + C_2 \tag{7}$$

In Eq. (6),  $Y'(p)$  is the derivative of the curve of deflection. Both the constants of integration,  $C_1$ , and  $C_2$ , can be calculated as shown below. When  $p$  equals 1,  $Y'(1)$  will be 0 and  $Y(1)$  will be 0. Without any kind of derivation, the values for both constants are as follows

$$C_1 = \frac{1}{2} \frac{m - 2}{(1 - m)^2} \tag{8}$$

Both the constants,  $C_1$  and  $C_2$ , are now substituted back in Eq. (6) and (7).

Now we wish to uncover the slope and deflection at the free end. Since  $p=0$  at the free end, we have

$$Y'(0) = \frac{1}{2m} \tag{9}$$

$$Y(0) = \frac{1}{2} \frac{3 + 2 \ln(m) - 4m + m^2}{(1 - m)^2} \tag{10}$$

By multiplying Eq. (9) and (10) by  $PL/EI_0$ , we can ascertain the true slope and deflection. This gives us

$$Y'(0) = \frac{1}{2m} \frac{PL}{BI_0} \tag{11}$$

$$y(0) = \frac{1}{2} \frac{3 + 2 \ln(m) - 4m + m^2}{(1 - m)^2} \frac{PL}{BI_0} \tag{12}$$

Now for a rectangular beam,  $h_0$  will be equal to  $h$  or  $m$  will be 1. This will give us

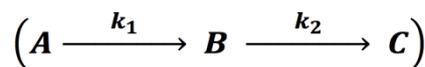
$$y'(0) = \frac{PL}{2BI_0} \tag{13}$$

$$y(0) = \lim_{m \rightarrow 1} \frac{1}{2} \frac{3 + 2 \ln(m) - 4m + m^2}{(1 - m)^2} \frac{PL}{BI_0} = - \frac{PL}{3BI_0} \tag{14}$$

### 3. FIRST ORDER IRREVERSIBLE SERIES REACTION

Mathematics also has numerous applications in chemical engineering; refer [6-8] for examples. [6] Contains situations solved and modeled by Maple. [7], on the other hand, contains diverse applications of mathematics in chemical engineering including kinetic modeling, computational fluid dynamics, control problems, and certain diffusion/reaction questions. [8] Is a case study which explores the planning of biotreatment systems. Other books [10-11] discuss thermodynamic properties and contain an analysis of various equations of state.

The example considered below has been extracted from [6]. Consider the following first-order irreversible series reaction  
The primary equations for this reaction are



$$\frac{dC_A}{dt} = -k_1 C_A \tag{15}$$

$$\frac{dC_B}{dt} = k_1 C_A - k_2 C_B \tag{16}$$

Where the rate constants are  $k_1$  and  $k_2$  and initial conditions are  $C_A(0) = 1$  mol/l,  $C_B(0) = 0$ , and  $C_C(0) = 0$ . The time-specific concentration of species  $C(t)$  can be modelled by the material balance as follows

$$C_C = 1 - C_A - C_B \tag{17}$$

Through the use of Maple, Eq. (15) and (16) are solved as

$$C_A = e^{-k_1 t} \tag{18}$$

$$C_B = \frac{k_1(e^{-k_1 t} - e^{-k_2 t})}{k_2 - k_1} \tag{19}$$

For illustrative purposes, we can take  $k_1$  as 2 and  $k_2$  as 3. This gives us

$$C_A = e^{-2t} \tag{20}$$

$$C_B = 2e^{-2t} - 2e^{-3t} \quad (21)$$

$$C_C = 1 - 3e^{-2t} + 2e^{-3t} \quad (22)$$

Here as the value of  $t$  reaches closer to infinity, the values of both  $C_A$  and  $C_B$  reach closer to 0 while  $C_C$  will tend to 1. This observation, however, can even be observed when the value of  $t$  reaches 5. As  $t$  approaches 5, both  $C_A$  and  $C_B$  reduce drastically to almost zero while  $C_C$  approaches very close to 1.

Figure 2 displays how the graph of the concentrations varies over time.

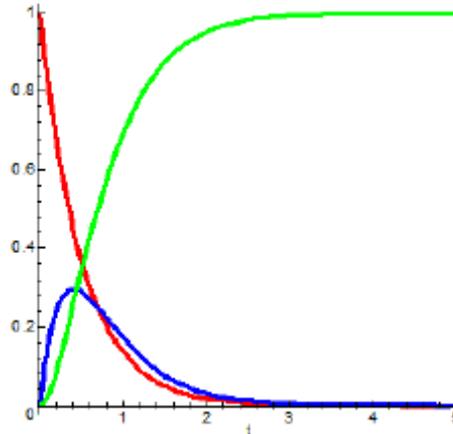


Fig. 2: Displaying how concentrations of  $C_A$  (red),  $C_B$  (blue) and  $C_C$  (green) change over time

Now, we move on to evaluating the reaction equations. First, we are interested in figuring out the time-point when  $C_A$  and  $C_B$  have a similar concentration. To achieve this we equate Eq. (20 and (21) and get

$$e^{-k_2t} = \frac{k_1(e^{-k_1t} - e^{-k_2t})}{k_2 - k_1}$$

This can now be solved to get

$$t^* = \frac{\ln \left[ \frac{k_1}{2k_1 - k_2} \right]}{k_2 - k_1} \quad (23)$$

Substituting the equation for  $t^*$  to Eq. (18) or (19) will give

$$C_A^* = C_B^* = \left[ \frac{k_1}{2k_1 - k_2} \right]^{\frac{k_1}{2k_1 - k_2}} \quad (24)$$

Now with  $k_1$  as 2 and  $k_2$  as 3, we get the result  $t^* = \ln(2)$  and  $C_A^* = C_B^* = 1/4$

The next question arises now: what if  $k_1 = k_2$ ? When we substitute  $k_1 = k_2$  in Eq. (19) we obtain a 0 divided by 0 form which is not determinable. However, checking for the limit of  $C_B$  as  $k_2 \rightarrow k_1$  gives

$$C_B = \lim_{k_2 \rightarrow k_1} \frac{k_1(e^{-k_1t} - e^{-k_2t})}{k_2 - k_1} = k_1 t e^{-k_2t} \quad (25)$$

So, when  $k_1$  is equal to 2 we have

$$C_A = e^{-2t}, \quad C_B = 2te^{-2t}, \quad C_C = 1 - e^{-2t} - 2te^{-2t}$$

For what value of  $t$  will  $C_B$  be maximum? Differentiation of eq. (19) with respect to  $t$  and equating the result to zero will give

$$\frac{dC_B}{dt} = \frac{k_1(k_1 e^{-k_1t} - k_2 e^{-k_2t})}{k_1 - k_2} = 0$$

Which further gives

$$t_{max} = \frac{\ln \left[ \frac{k_1}{k_2} \right]}{k_1 - k_2} \quad (26)$$

Substituting  $t_{max}$  in Eq. (19) gives

$$C_{B\ max} = \frac{k_1}{k_1 - k_2} \left[ \left( \frac{k_1}{k_2} \right)^{-k_2/(k_1-k_2)} - \left( \frac{k_1}{k_2} \right)^{-k_1/(k_1-k_2)} \right] \quad (27)$$

Substituting  $k_1$  as 2 and  $k_2$  as 3 returns values of  $t_{max}$  as 0.4055 and  $C_{B, max}$  as 0.2963. In the case that  $k_1$  equals  $k_2$ , we are able to ascertain the limit of  $C_{B, max}$  due to the similarity of  $k_2$  and  $k_1$ . However, since that assumption might not hold in all cases, it is simpler to just differentiate Eq. (25) in regards to variable  $t$  and then equate the resultant to 0. This gives us,

$$\frac{dC_B}{dt} = k_1 e^{-k_1 t} (1 - k_1 t) = 0$$

Which after solving yields  $t$  as  $1/k_1$ . Putting this expression of  $t$  in Eq. (25) gives the result  $C_{B, max} = 1/e$ . It is also observed that as  $k_1 = k_2$ , the highest concentration for CB is completely unlinked to the rate constants. Furthermore, the time to reach that concentration is found to be inversely proportional with rate constant  $k_1$  [6].

It is a little more tricky to evaluate first order reversible series. For each of the three concentrations, there are three linear equations (first order differential) that need to be solved first. However, even such questions can be solved using Maple without major difficulties (Refer to 6 for solution)

#### 4. ILLUMINATION PROBLEM

**This problem goes as:** A courtyard is provided illumination through two lamps. Here the illumination power is given by  $P_1$  while the heights of the lamps by  $h_1$  (Refer to figure 3). The Euclidean coordinates of the lamps assumed to be on a plane, are  $(0, h_1)$  and  $(s, h_2)$ . Here  $s$  is the horizontal distance between the two lamps. Now let us assume there is a point X which is at a distance of  $x$  from the left lamp, i.e. has coordinates  $(x, 0)$ , between the two lamps. Our objective is to locate the point X attracting the minimum illumination from both the lamps. Within this problem, the illumination power is in kilowatts (kW) while the heights and distance between the lamps in meter (m). This problem will also be solved using Maple.

The used illumination problem is extracted from [12]. Within this set-up, a courtyard is lit by two lights. P

From figure 3 we have,

$$r_1^2 = h_1^2 + x^2, \quad r_2^2 = h_2^2 + (s - x)^2$$

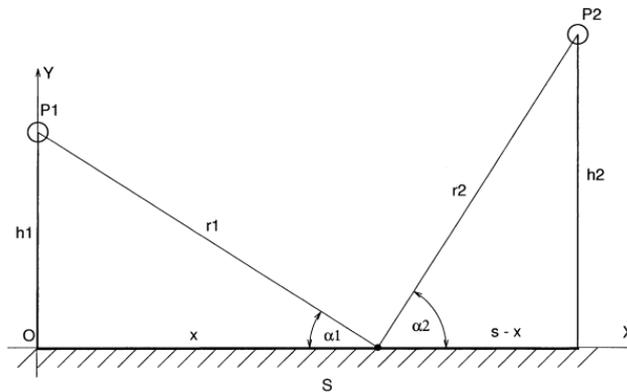


Fig. 3: Illumination problem

As given in [12], the intensity of the light from both the lamps at any point, X, is given as

$$I_1(x) = \frac{P_1}{r_1^2} = \frac{P_1}{h_1^2 + x^2}, \quad I_2(x) = \frac{P_2}{r_2^2} = \frac{P_2}{h_2^2 + (s - x)^2}$$

The formula for the illumination from both lamps at the point x is  $I_i(x) = I_i(x) \sin(\alpha_i)$  so we have

$$II_1(x) = \frac{P_1 h_1}{\sqrt{(h_1^2 - x^2)^3}}, \quad II_2(x) = \frac{P_2 h_2}{\sqrt{(h_2^2 + (s - x)^2)^3}} \quad (28)$$

To provide greater ease in typing, the symbols in the above equations are modified where P1 is written as p, P2 as Q, h1 as a and h2 as b. Total illumination at any point X then becomes

$$C(x) = II_1(x) + II_2(x) = \frac{pa}{\sqrt{(a^2 + x^2)^3}} + \frac{qb}{\sqrt{(b^2 + (s - x)^2)^3}} \quad (29)$$

Here,  $C(x)$  will be least at the point when  $dC(x)/dx$  equals 0.

Therefore,

$$\frac{dC(x)}{dx} = -\frac{3pax}{(a^2 + x^2)^{3/2}} + \frac{3qb(s-x)}{(b^2 + (s-x)^2)^{3/2}} = 0 \quad (30)$$

Maple, however, is unable to handle this general solution. To make the question compatible with its software, the radical sign is removed by squaring the expression. This gives

$$\frac{p^2 a^2 x^2}{(a^2 + x^2)^2} = \frac{q^2 ab^2 (s-x)^2}{(b^2 + (s-x)^2)^2} \quad (31)$$

By simplifying and expanding the above equation, we receive a degree-12 polynomial. This, again, is out the capabilities of Maple. For instance, let us insert  $p$  as 1 kW,  $q$  as 2 kW,  $a$  as 4m,  $b$  as 5m and  $s$  as 10m.

Replacing Eq. (31) by these assumed values gives after expansion of the result

$$F(x) \equiv 21x^{12} - 100x^{11} - \dots - 52428000x + 2621440000 = 0 \quad (32)$$

Due to the purpose of conciseness, the complete expression cannot be shown here.

Calculating mathematical solutions of Eq. (32) will yield three real roots when restricting the values of  $x$  between 0 and 10, namely 4.4112, 0.1541 and 9.9111. Among these values, the one at which  $C(x)$  is the least can be ascertained by evaluating the first derivative of  $F(x)$ . If it is greater than 0, then  $C(x)$  will be the least. In this case, it is 4.4112 meaning 441 centimeters along the left lamp. Using this value in Eq. (29) yields the minimum illustration as 0.0427.

Eq. (30) can also be directly solved without converting it to the general form. Here, substituting  $p$  for 1,  $q = 2$ ,  $a = 4$ ,  $b = 5$  and  $s = 10$  to Eq. (30) will yield

$$-\frac{12x}{(10 + x^2)^{3/2}} + \frac{30(10 - x)}{(25 + (10 - x)^2)^{3/2}} = 0$$

The solutions to this non-linear equation are 0.1541, 4.4112 and 9.9111, similar to the previous results.

## 5. CONCLUSION

Through the course of this paper, three different real-life instances of application of mathematics in distinct engineering fields have been displayed. Each of these problems was solved using the Maple software. None of these problems required advanced mathematics to solve and it is reasonable to expect any engineering student proficient in calculus can solve them without much difficulty. These examples were meant to motivate engineering students to develop an interest in and a greater understanding of mathematics.

## 6. REFERENCES

- [1] R. M. Corless, *Essential Maple*, Springer-Verlag, New York, 1995.
- [2] Gere, J.M. and Timoshenko, S.P., *Mechanics of Materials, Third SI Edition*. Dordercht: Springer Science+Business Media, 1991.
- [3] Popov, E., *Engineering Mechanics of Solids*. New Jersey: Prentice-Hall, 1990.
- [4] J. E. Connor, J.E. and Faraji, S., *Fundamentals of Structural Engineering*. Berlin Heidelberg: Springer-Verlag, 2012.
- [5] Hjelmstad, K.D., *Fundamentals of Structural Mechanics, Second Edition*. New York: Springer-Verlag, 2005.
- [6] White, R.E. and Subramaniam, V.R., *Computational Methods in Chemical Engineering with Maple*. Springer-Verlag. Berlin Heidelberg, 2010.
- [7] Keil, F., Mackens, W., Vo, H. And Werther, J., *Scientific Computing in Chemical Engineering*. Springer-Verlag, Berlin Heidelberg, 1996.
- [8] Caldwell, J. and Ram, Y.M., *Mathematical Modelling*, Springer Science+Business Media. Dordercht, 1999.
- [9] Braun, M., *Differential Equations and Their Applications*, Springer Science+Business Media. New York, 1993.
- [10] Jacobsen, R.T., Penoncelo, S.G. and Lemmon, E.W., *Thermodynamic Properties of Cryogenic Fluids*. Springer Science+Business Media, New York, 1997.
- [11] Reid, R.C., Prausnitz, J.M. and Poling, B.E., *The Properties of Gases and Fluids*. McGraw-Hill Inc., New York, 1987.
- [12] Gander, W. And Hrebicek, J., *Solving Problems in Scientific Computing Using Maple and MATLAB®*. Springer-Verlag, Berlin Heidelberg, 2014.
- [13] K. M. Heal, M. L. Hansen, and K. M. Rickard, *Maple V Learning Guide*. Springer-Verlag, New York, 1998.